ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

As always, candidates are to be encouraged to read the rubric on the front of the examination paper. Not enough attention is placed on the accuracy of working through a solution and the final answer. Too many candidates lost marks due to either premature rounding during working the solution or not giving the final answer to the correct level of accuracy. It is also essential that candidates ensure that they have satisfied the demands of the question, e.g. giving an answer in an exact form or a specified form.

In a question where a candidate is expected to show a certain result, it is essential that each step of the working required is shown in order to gain full marks.

General comments

There were many candidates who performed well, showing a good understanding of the syllabus objectives and producing clear and well thought out solutions. However, there were also many candidates who had clearly not prepared enough to make a good attempt at the paper.

It is recommended that candidates request additional paper if they find that they do not have enough room to complete their solution in the space provided. This is preferable to solutions which are cluttered and difficult to follow. Candidates are also requested not to overwrite solutions originally done in pencil in ink as this makes them difficult to read.

Comments on specific questions

Question 1

(i) With only one mark available, the solution given was either correct or incorrect. It was expected that candidates draw two separate sets, labelled appropriately, with no overlap. Anything else was deemed to be incorrect.

(ii) Many candidates were unsure of the meaning of the symbol $\subset$. This resulted in many incorrect answers, often a Venn diagram showing the set C containing both sets A and B. Full marks were awarded for a Venn diagram showing set C with no overlap with either set A or set B. There were several acceptable versions due to the fact that sets A and B could be regarded as having an intersection, not having an intersection or one being a subset of the other. One mark was available for the few candidates who assumed that set C was the entire region representing $(A \cup B)'$.

Question 2

Very few completely correct solutions were seen as there was a common error which many candidates made. Most candidates realised that the value of *a* was related to the amplitude and gained a mark for the statement or implication that $a = 4$ Most candidates then assumed, incorrectly, that $b = \frac{\pi}{3}$. However, the majority of candidates correctly attempted to find the value of *c* by substituting in their value of *a* and of *b* correctly into the equation given, using the coordinates of the given point. This usually merited a method mark. Other less common errors included using $b = 3$ or making arithmetic slips in the calculation of *c* after correct substitution, with $c = 2$ being the most common error.

Answer: $a = 4,\ b = 6,\ c = -2$

Question 3

(i) Marks were awarded on the basis of one mark for each correct simplified term. Candidates could opt for use of Pascal's triangle or make use of the binomial expansion given on the formulae page. There were many correct solutions, but candidates lost marks due to sign errors and omitting to consider the power of 4 when finding the third term. Very few candidates did not have terms in x^2 and x^4 for the second and third terms in their expansions.

(ii) It is evident that many candidates do not understand what is meant by 'the term independent of *x*', even though requests of this type have been made in the past. Most candidates were able to obtain a mark for the expansion of $\left(\frac{1}{x} - \frac{3}{x^2}\right)^2$ although some candidates attempted use of the binomial expansion for this relatively simple squaring – usually with no success. For candidates that did realise that use of the expansion of $\left(\frac{1}{x} - \frac{3}{x^2}\right)^2$ together with their result from part **(i)** was needed and thus there were only two terms that needed to be considered, there were errors with signs. If a candidate had made errors in part **(i)**, all the marks in part **(ii)** were available on a correct follow through basis.

Answers: **(i)** $32 - 20x^2 + 5x^4$ **(ii)** 25

Question 4

It was essential that a correct formula for either the differentiation of a quotient or an appropriate product was used. Candidates were able to obtain marks even if they were unsure of the correct differentiation of a logarithmic term, provided they did not assume that the derivative of a logarithmic term also contains a logarithmic term. It is suggested that in questions of this type, a numerical substitution is done immediately after application of the correct formula and before any algebraic simplification. Errors in algebraic simplification cost many candidates accuracy marks, whereas numerical simplification would have probably yielded greater accuracy. Candidates are requested to note that the form of the answer is given in the question itself and that if they find that they do not have a solution which has this form then a check on previous working should be made.

Answer: $\frac{6}{35} - \frac{4}{25}\ln 14$

Question 5

(i) Many correct solutions were seen, with candidates realising that the equation needed was of the form $\lg y = mx + c$. Most candidates were able to obtain a mark for the calculation of the gradient, but some errors were made when attempting to find the value of *c*, usually due to the incorrect use of logarithms. For those candidates who did not have the required form, a mark for the gradient was usually the only mark available.

(ii) It was evident that many candidates did not know exactly what was required, so there were very few correct solutions. Many of those candidates who did make a partially correct attempt, realised that b was equivalent to the coefficient of their x term in part **(i)** and were able to get credit for this. Most problems occurred with the calculation of A. Of the two possible methods that could be used for this question, the most successful method was re-writing the given equation in appropriate logarithmic form and comparing this with the answer obtained in part **(i)**. The other method of writing the answer to part **(i)** in index notation involving 10, was less successful.

Answers: **(i)** $\lg y = 0.42 - 0.2x$ **(ii)** $y = 2.63\left(10^{-0.2x}\right)$

Question 6

Many candidates appear to have problems with the notation used when dealing with functions. It is clearly a topic which needs clarification, consolidation and practice for many.

(i) Very few candidates realised that the range of $\ln x$ is the set of all real numbers. An appreciation of the graph of $y = \ln x$ would have helped with this question. It was evident that even for incorrect ranges, candidates are not using acceptable notation to denote the range. The use of y, f or $\mathrm{f}(x)$ are all acceptable for use when stating a range, however the use of x only is not.

(ii) Similar comments to those made in part **(i)** apply to part **(ii)** as well, although there were more correct responses to this part. More candidates were able to relate the shape of the function to the range and realise that 3 was a critical value. However, mistakes in notation were common as were mistakes in the type of inequality sign used.

(iii) Most candidates were able to obtain a mark for the correct order which was usually implied by the appearance of 35 and less often by e^x. Many candidates did not know the inverse of $\ln x$, but were usually able to obtain a mark for 35. The question asks for the exact value and provided e^{35} was seen with subsequent working, then the subsequent working was ignored and full marks given. It was noted however that in subsequent working, some candidates who made use of their calculators to evaluate e^{35}, did not or could not read and interpret their calculator display correctly with 1.57 being a common response rather than the correct 1.57×10^{15}.

(iv) Most candidates were able to obtain a method mark for an attempt to find $\mathrm{g}^{-1}(x)$. There were a few arithmetic slips but other common errors included: $\mathrm{g}^{-1}(x) = \frac{\sqrt{x-3}}{2}$ with a misplaced square root sign, $\mathrm{g}^{-1}(x) = \pm\sqrt{\frac{x-3}{2}}$ and $x = \sqrt{\frac{y-3}{2}}$. All these answers would have gained one mark for an implied correct method. The use of y rather than $\mathrm{g}^{-1}(x)$ was also acceptable. Many candidates forgot that they were also required to state the domain of the function. Of those that did not, there were few correct responses, with errors in the type of inequality sign used being the most common.

Answers: **(i)** $\mathrm{f} \in \mathbb{R}$ **(ii)** $\mathrm{g} > 3$ **(iii)** e^{35} **(iv)** $\mathrm{g}^{-1}(x) = \sqrt{\frac{x-3}{2}}$, $x > 3$

Question 7

(i) The great majority of candidates were able to write down two correct unsimplified equations having made correct use of both the factor theorem and the remainder theorem. Subsequent errors in simplification and the solution of two simultaneous equations meant that many incorrect values for *a* and for *b* were seen. At this level, too many arithmetic errors are being made. It should also be noted that many candidates did not have enough space to write their answer out clearly and concisely. If this occurs, it is essential for candidates to ask for additional sheets so that they may be able to work in a more orderly fashion and produce more accurate work. Again, candidates should take note of the wording of the question. They are told that *a* and *b* are both integers, so obtaining solutions to simultaneous equations that are not integers should have alerted them to the fact that there must be errors in their work. Some checking of simplification could have then resulted in a correct solution.

(ii) This part was poorly done by most. Many did not understand the question and started using a substitution of $x = 5$. Others attempted the use of the quadratic formula on a cubic equation. Others mistakenly thought that the factor $2x - 1$ was involved. Even if the values from part **(i)** were incorrect, candidates could gain a mark for the solution $x = 0$ which is independent of the value of *a* and of *b*. Some candidates lost this solution by dividing through by *x*. However, a mark was available if the quadratic formula was used on a correct equation provided an answer in exact form was given. Too many candidates relied on their calculator to give a decimal answer rather than take note of the demand of the question.

Answers: **(i)** $a = 12,\ b = -17$ **(ii)** $x = 0,\ -\frac{1}{3} \pm \frac{\sqrt{55}}{6}$

Question 8

This question was the least popular and the most poorly done on the paper. It is evident that candidates have difficulty with this topic. There were many candidates who made no attempt at this question at all.

(i) Of the candidates that attempted this question, many were able to gain a mark for calculating the angle *AB* made with the river bank, although incorrect rounding in some cases meant that this mark was lost. At this point many candidates thought they had answered the question stating that $67.4° \approx 65°$. A triangle needed to be drawn showing the line which the man must point his boat at an angle of, e.g. θ, upstream to the bank. The sine rule may then be used on the triangle formed using this line, the direction of line *AB* and direction of the current in order to find the appropriate angle. Candidates should be aware that the work required for a mark allocation of four marks is likely to be more than just finding one angle.

(ii) Many candidates were able to obtain two marks for finding the distance the boat had to travel, but few found the correct resultant velocity and were thus unable to find a correct time. Some completely correct solutions were seen, with correct alternative methods being acceptable.

Answers: **(ii)** 26.5 s

Question 9

(a) The topic of graphs in the syllabus objective of Kinematics is problematic for many candidates. Very few correct solutions were seen, with many of them just mirroring the shape of the original graph. It was intended that candidates realise that in each leg of the journey constant velocity was involved and this is represented by a straight line of gradient zero on a velocity time graph. It was expected that candidates draw a straight line joining the points $(0, 5)$ and $(10, 5)$, and another straight line joining the points $(10, 0.5)$ and $(30, 0.5)$.

(b) (i) Most candidates were able to gain the mark available for substitution of $x = 0$ and subsequent result of 3.

(ii) Many candidates attempted to differentiate with varying levels of success. It was evident that some candidates do not appreciate the effect of differentiation upon exponential functions and are also unable to solve equations involving the exponential function. Many candidates lost accuracy marks in this part by not giving the answer to the correct level of accuracy, 0.46 or even 0.5 being offered as solutions to an otherwise correct equation involving an exponential.

(iii) Candidates needed to realise that integration was needed to find the distance required. Again, poor skills in dealing with an exponential function and calculus meant that some candidates were unable to achieve. Those candidates that introduced limits and integrated definitely were usually more successful than those who chose indefinite integration as they often did not find the value of the arbitrary constant. Again, accuracy marks were often lost due to not giving the final answer to the required level of accuracy.

Answers: **(b)(i)** 3 **(ii)** 0.461 s **(iii)** 0.738 m

Question 10

(i) Most candidates realised that, for a single mark, all that was needed was to make use of the relationship between the given arc length and the given radius.

(ii) Candidates had to realise that they needed to find the lengths *BC* and *BD* in order to make any progress with finding the perimeter of the given shape. Unfortunately, many candidates mistakenly assumed that the triangle *ABC* was equilateral and used a length or 5 cm for *BC*. The idea of asking the candidates to find angle *BAC* in part **(i)** was to help them realise that this was not the case. As a result of this, fewer than expected correct solutions were seen.

(iii) Even if a correct value for *BC* had not been found in part **(ii)**, candidates were still able to gain marks for the area of the segment *BECD*. A surprising number of candidates had difficulty with this part (as well as part **(ii)**) even though questions of this type are tested frequently. Errors involving premature approximation meant that some candidates, although using correct methods, were unable to score the final accuracy mark.

Answers: **(ii)** 15.3 cm **(iii)** $9.58 \leqslant \text{area} \leqslant 9.62$

Question 11

(a) Too many candidates were either unsure of how to deal with cotangent, or mistakenly thought that $\cot(\phi + 35°) = \cot\phi + \cot 35°$. For these candidates, no marks were available. Many candidates however, were able to reach a value of 21.8°, which indicated a correct use of cotangent. Many candidates were then able to obtain the solution in the second quadrant, but fewer candidates were able to obtain both solutions. Candidates are unwilling to consider the fact that angles greater than 360° have to be considered in questions of this type.

(b) (i) Most candidates were able to gain some marks in this question, with many gaining full marks. It is essential that each step of working is shown in order to gain full marks, so identities must be shown. Candidates who chose to omit the angle throughout their work were penalised by loss of the final accuracy mark. Likewise, candidates who did not use brackets in a correct manner were also penalised by the loss of the final accuracy mark.

(ii) Very few candidates scored full mark for this question. The problem appeared to be the inclusion of negative angles in the required range. This has not been tested a great deal in the past but is covered by the syllabus. Many candidates treated the multiple angle incorrectly and as a result were unable to gain any marks. Others did not give their answers in terms of π, but were often able to gain a method mark. Usually candidates only gave the positive angle in the given range. Clearly this is a topic that needs to be worked on.

Answers: **(a)** 166.8°, 346.8° **(b)(ii)** $-\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{4\pi}{9}$

ADDITIONAL MATHEMATICS

Paper 4037/13
Paper 13

Key messages

Candidates should take care when copying answers for use in further parts of a question.

In questions where the use of radians is indicated, candidates should be encouraged to use radians in calculations rather than converting to degrees.

General comments

The paper provided a good range of responses showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues. Candidates made good use of the space provided on the paper and responses were generally straightforward to mark with a good standard of presentation. There was some carelessness, particularly when candidates copied their own figures to use in a further part of a question.

Comments on specific questions

Question 1

Most candidates made a correct start using $y = 2(1 + \tan^2\theta)$. However, few candidates went on to express $\tan\theta$ in terms of x and hence obtain an expression for y in terms of x.

Answer: $y = 2((x+5)^2 + 1)$

Question 2

Candidates should be aware that the equation for a curve is found by integrating the gradient function and then finding the constant of integration. There seemed to be some confusion with the method for finding the equation of a straight line and many candidates attempted $y = mx + c$ using the gradient function as if it was a numerical gradient. Some good responses were seen but a high proportion of candidates did not recognise that integration was required.

Answer: $y = 2e^{5x} + 3x + 7$

Question 3

All candidates knew that use of the discriminant was required with many of them making the correct substitutions using $a = k$, $b = 3$ and $c = k - 4$. Although a good number of candidates obtained correct critical values, most encountered difficulty with the manipulation required for obtaining correct inequalities for k. Candidates using a calculator to find critical values tended to assume factors of $(2k - 9)(2k + 1)$ rather than $(9 - 2k)(2k + 1)$ and hence mishandled the inequalities. Candidates should be advised to return to the original inequality and check their range using values of k or to make a sketch relevant to the original inequality. Candidates were tempted to combine their answer into a three-part inequality which was not appropriate here.

Answer: $k < -\frac{1}{2}$, $k > \frac{9}{2}$

Question 4

Most candidates obtained $a = 3$ but it was not unusual to assume that b was equal to $\frac{\pi}{4}$. Candidates should be aware that b is obtained by dividing 2π by the period. Some responses were confused by attempts to convert to degrees rather than using radians, but most candidates who had stated a value for b made correct substitutions in an attempt to find c.

Answer: $a = 3$, $b = 8$, $c = 4$

Question 5

(i) Most candidates made a good attempt at integration and knew to increase the power by one. Candidates often divided by $\frac{2}{5}$ but division by 7 was often omitted. Mistakes in simplification were not uncommon and led to a loss of marks in the next part.

(ii) Candidates who had obtained an integral of a suitable form usually subtracted the correct way around. Misunderstandings concerning the simplification of $\frac{5}{14}(7a-10)^{\frac{2}{5}}$ and $\frac{5}{14}(7\times 6-10)^{\frac{2}{5}}$ were often evident. Candidates had to obtain the value of a from a correct order of operations and should be aware that $\frac{5}{14}$ cannot be 'taken into the bracket' and that $(7\times 6-10)^{\frac{2}{5}}$ must be evaluated before multiplication by $\frac{5}{14}$. The penultimate step of raising 9 to a power of $\frac{5}{2}$ proved to be difficult for those whose order of operations had been correct to that point, particularly if they had not obtained $\frac{5}{14}$ in the first part. The answer required was exact; $\frac{253}{7}$ and $36\frac{1}{7}$ were acceptable but not a rounded decimal.

Answers: **(i)** $\frac{5}{14}(7x-10)^{\frac{2}{5}}$ **(ii)** $\frac{253}{7}$

Question 6

(i) Nearly all candidates obtained a correct gradient. Not all appreciated that the given coordinates were (x^2, ln$y$) and that the substitutions made to find q should be of the form $p(0.4) + q = 2.4$. It was not sufficient just to find p and q; an equation for ln$y$ had to be obtained with correct values of p and q.

(ii) Candidates were clearly unsure how to relate this question to the previous part as it was often omitted. Candidates should understand the relationship between the natural logarithm and the exponential function. They would also benefit from practice in manipulation of indices. Few candidates appreciated that y was in fact a product of powers of e.

Answers: **(i)** $\ln y = -\frac{5}{2}x^2 + 2.9$ **(ii)** $y = 18.2z^{-\frac{5}{2}}$

Question 7

(i) Most candidates showed a good knowledge of how to apply the formula provided for the binomial expansion. However, the second and third terms were sometimes incorrectly simplified or not simplified. Mistakes were made when candidates used $\left(+\frac{x^2}{4}\right)$ or $\left(-\frac{x}{4}\right)$ rather than $\left(-\frac{x^2}{4}\right)$ or simplified $\left(-\frac{x^2}{4}\right)^2$ to $-\frac{x^4}{16}$ or to $\frac{x^4}{4}$.

(ii) Care had to be taken with the expansion of $\left(\frac{1}{x}+x\right)^2$ and the middle term was sometimes missing or incorrect. Candidates showed a good understanding of combining terms to obtain a term in x^2 with the term from $15x^4$ and $\frac{1}{x^2}$ being occasionally omitted. Most candidates scored some marks with many either fully correct or correct on following through.

Answers: **(i)** $64-48x^2+15x^4$ **(ii)** -17

Question 8

(i) Most candidates realised that the product rule had to be used and demonstrated a good understanding of its use. $(3x-1)^{\frac{5}{3}}$ was differentiated well with occasional loss of the multiple of 3. Candidates had to use $(3x-1)^{\frac{5}{3}}=(3x-1)(3x-1)^{\frac{2}{3}}$ to factorise to get an answer in the given form but many candidates were hampered by incorrect attempts at simplification before this stage. Influenced by the form given in the question, many gave $(3x-1)^{\frac{2}{3}}(5x-20)$ as their final answer rather than attempting factorisation.

(ii) This question was often omitted. Candidates who did attempt the question usually took a correct approach and did well, substituting $x = 3$ in an expression of the correct form or in their original product rule expression and multiplying by h.

Answers: **(i)** $(3x-1)^{\frac{2}{3}}(8x-21)$ **(ii)** $12h$

Question 9

(a) (i) Nearly all candidates recognised this as a permutation and answered correctly.

(ii) Most candidates adopted a correct strategy and gave a correct answer.

(iii) Many candidates appeared to have difficulty forming a plan to approach this question. Most multiplied 4! by 2 or used an equivalent in additions of products but few candidates realised that there were four alternatives for the first digit once 6 or 8 had been used in the last position.

(b) (i) Nearly all candidates recognised this as a combination and answered correctly.

(ii) Candidates had to add the number of combinations from two cases 'with twins' and 'without twins' and realise that in each case the other children were being chosen from the remaining thirteen. Candidates had difficulty forming this plan and would benefit from practice in this type of question. Many candidates considered just one of these cases.

Answers: **(a)(i)** 720 **(ii)** 240 **(iii)** 192 **(b)(i)** 6435 **(ii)** 3003

Question 10

In both parts of this question candidates showed a good understanding of matrix multiplication. However, candidates would benefit from a more thorough knowledge of matrix algebra when using the inverse matrix to solve problems and should realise the importance of the correct order of multiplication.

(a) The most common and successful approach was to multiply **A** and **B** and then form and solve two simultaneous equations. Many candidates scored full marks. Those who attempted the alternative method using $\mathbf{A} = \mathbf{ABB}^{-1}$ were not as successful. Candidates who did not form a plan tended to obtain a correct inverse matrix but performed $\mathbf{B}^{-1}\mathbf{AB}$.

(b) (i) Finding the inverse matrix was well attempted. Marks were lost when candidates correctly calculated –17 but then chose to use +17 in their inverse.

(ii) Most candidates correctly used their inverse and pre-multiplied to gain the method mark and one or two of the accuracy marks. Unsuccessful candidates multiplied in the incorrect order not realising that $\mathbf{Z} = \mathbf{X}^{-1}\mathbf{XZ} = \mathbf{X}^{-1}\mathbf{Y}$.

Answers: **(a)** $a = 4$ $b = -2$ **(b)(i)** $-\frac{1}{17}\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$ **(ii)** $-\frac{1}{17}\begin{pmatrix} 19 & 2 \\ 8 & 8 \end{pmatrix}$

Question 11

Candidates generally showed a good understanding of what had to be done in this question. However, they should be advised to use length of arc = $r\theta$ and area of a sector = $\frac{1}{2}r^2\theta$, where θ is an angle measured in radians. Conversions between degrees and radians meant that those who tried to use formulas using angles in degrees tended to lose accuracy.

(i) Most candidates used $r\theta = 14.8$ to obtain a correct answer.

(ii) Most candidates used $\frac{1}{2}r^2\theta = 21.8$ successfully but there were a significant number who confused the area of the sector with the area of the triangle.

(iii) Finding the angle *BOC* was of key importance for success in both of parts **(iii)** and **(iv)** and candidates who made mistakes calculating that angle lost several marks. A common mistake in calculation of the angle was to subtract 1.48 and 0.436 from π rather than 2π. Some candidates forgot to divide by 2 and others were handicapped by an incorrect answer in part **(ii)**. Small errors also came from attempts to change between radians and degrees and from premature rounding of 2π and answers to parts **(i)** and **(ii)**. Most candidates knew they had to calculate the length *BC* and most used the cosine rule. Nearly all candidates formed a correct plan.

(iv) A variety of methods were seen but the most common was to add the two sector areas to the twice the area of the triangle *BOC*. Most candidates obtained the two sector areas, but some chose to calculate a sector area using their incorrect angle from part **(ii)** rather than using 21.8 given in the question. The most successful method for the area of the triangle was $\frac{1}{2} \times 100 \sin BOC$ but those who used an incorrect angle *BOC* in their calculation lost marks.

Answers: **(i)** 1.48 **(ii)** 0.436 **(iii)** 54.7 **(iv)** 178

Question 12

(i) This question was answered well with candidates demonstrating that they could use their knowledge to form a strategy to tackle a long, less structured question. Most candidates embarked on a correct method to find the coordinates of *P* and *Q*. Not all realised that the mid-point of *PQ* was required. Most knew that a perpendicular gradient was required and used a correct method to find it. However, a significant number of candidates did not realise that $y = 2x + 1$ was the equation of *PQ* and made a fresh start using their coordinates for *P* and *Q* rather than obtaining the gradient by inspection. A good number of candidates were able to reach the end of the question and demonstrate correctly by a variety of methods that $\left(-10, \frac{23}{8}\right)$ lay on the perpendicular bisector.

(ii) This question was often omitted even by candidates who had made a good attempt at the first part. A clear mental picture of the problem was required to form a strategy and candidates would have been helped in their approach by a carefully drawn diagram. Those who started by substituting $x = 0$ into the two straight line equations to find R and S usually proceeded well. However, care was not always taken in using the straight-line equation found in part **(i)** and fresh starts were often made to find both equations. Sign errors and copying errors were common in this question. Most candidates used a matrix method and those who used (0, 1), $\left(0, -\frac{17}{8}\right)$ and the correct mid-point in their matrix usually earned full marks. Attempts to use area $= \frac{1}{2}$ base $\times$ height were often marred by use of incorrect figures arising from a lack of understanding of the layout.

Answer: **(ii)** 1.95

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

Candidates should remember that the word 'hence' is an indication that subsequent work on a question needs to use what has just been established. Work which disregards this instruction will achieve no credit.

General comments

The presentation of work has improved this year and this is a much appreciated development. However, a problem does still exist when candidates write their response in pencil and later go over it in ink. This poses a problem as the script can become very difficult to read.

Comments on specific questions

Question 1

Many candidates scored the first mark for evaluating $\left(2+\sqrt{3}\right)^2$ and no more due to the fact that subsequent work contained superfluous algebra which did not separate constant and surd terms. Those who did realise that the way forward was to form a pair of simultaneous equations usually did so efficiently and successfully.

An alternative and swifter method was to divide the whole expression by $\left(2+\sqrt{3}\right)$ resulting in the equation $a\left(2+\sqrt{3}\right)+b=\sqrt{3}-1$ which is easier to deal with.

Answers: $a=1$, $b=-3$

Question 2

Some fully correct solutions were seen, but many did not get beyond an attempt to multiply both sides by $x^{0.5}+5x^{-0.5}$. A common error seen when attempting to multiply this expression was simply to multiply each power by 2 giving x^1+5x^{-1}. Those that did successfully multiply by $x^{0.5}+5x^{-0.5}$ were often unable to simplify the result to a three-term quadratic with an error being seen in at least one of the four terms. Weaker candidates often began by attempting to 'simplify' the left hand side by separately dividing the first and second terms of the numerator by the first and second terms of the denominator. A number of candidates attempted, with varying degrees of success, a substitution method to eliminate the fractional powers.

Answers: $x=2$ or $x=3$

Question 3

The best solutions used the two possible linear equations to find the values of 2 and –0.5. Even when these values were correctly found, getting the correct regions still proved to be a challenge with $x > -\frac{1}{2}$ often seen. Many candidates preferred to use a method which involved squaring but many often squared only the left hand side which resulted in an incorrect quadratic to solve.

Answers: $x > 2$ or $x < -\frac{1}{2}$

Question 4

Those candidates who removed logs successfully to obtain the equations $x + 4 = y^2$ and $7y - x = 16$ generally proceeded to successfully solve these equations usually by eliminating *x* and solving for *y*. Those who attempted to eliminate *y* rarely obtained the correct quadratic in *x*. A few candidates who solved for *y* wrote down the solution as *x* = rather than *y* = and as a result did not get correct pairs of solutions. The most common errors were to produce equations such as $x + 4 = 2y$ or $7y - x = 4$ displaying poor knowledge of rules of logarithms.

Answers: $x = 5,\ y = 3$ or $x = 12,\ y = 4$

Question 5

(i) This part was usually correct. However, a significant number of candidates incorrectly used permutations rather than combinations in all three parts.

(ii) A large number of candidates got a fully correct solution but a common mistake was to omit some of the combinations. The most efficient way to tackle the problem was to consider mystery and non-mystery only. If each book type was taken into consideration it was not uncommon to ignore, for example, 2 mystery with 2 romance or 2 mystery with 2 crime.

(iii) This part was more successfully attempted than part **(ii)** as each type of book had to be considered within each possible outcome and so there was less chance of an omission. There were some errors in the calculations and some candidates added their combinations rather than multiplying them.

Answers: **(i)** 210 **(ii)** 155 **(iii)** 105

Question 6

Those candidates who had a correct result in part **(i)** were usually able to correctly arrive at the given result in part **(ii)** with most providing sufficient working. A few candidates appeared to have tried to unsuccessfully work backwards from the result in part **(ii)**, with a few crossing out correct work in part **(i)** in an attempt to achieve the given expression.

In part **(iii)** most attempted to differentiate the given expression for *A* and set this equal to zero. A few found the square root rather than the cube root when attempting to find *x* and many did not find the associated value of *A*. Attempts at the second differential were often used to show that the value was a minimum although a factor of 2 was often missing from the second term. Many candidates scored 4 out of 5 for this part due to the missing value of *A.*

Answers: **(i)** $h = \frac{500}{\pi x^2}$ **(iii)** $x = 4.30$, $A = 349$

Question 7

(i) This was the most challenging question on the paper with very few candidates scoring full marks. Many did not appreciate that the given gradient was that of the normal and so did not even attempt to find the gradient of the tangent before doing any work with the function. Many decided to differentiate the given function by the quotient rule which was of no use at all. Those who did realise that integration was required often attempted to integrate the normal function and did so to the numerator and denominator separately. Those who did attempt to firstly find the gradient of the tangent often made a mistake with the signs or with their attempt to divide by $\sqrt{x}$. Others attempted to use the equation of a straight line through the given point with the gradient still as a function of *x*.

(ii) This part was slightly more successful, although there were very few completely correct solutions. It was not unusual for candidates who had not used the negative reciprocal in the previous part to do so here and as a result find a correct gradient for the tangent. There were many good attempts at finding the relevant equation through the point (4 , …).

Answers: **(i)** $y = 2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - 10$ **(ii)** $y = 5.5x - 20$

Question 8

(i) There were many correct answers to this part. Some candidates found $\mathbf{A}^{-1}$ first but then multiplied by 2 rather than 2^{-1}. Of those who found 2**A** first, most correctly found the inverse matrix, although a few, having correctly found the determinant found adj (**A**) rather than adj(**2A**).

(ii) A surprising number of candidates did not attempt to use matrices at all in this part or recalculated the inverse of a different matrix. They appeared not to have seen the word 'hence' in the question or had chosen to ignore it. Those who had used their matrix from part **(i)** usually correctly pre-multiplied $\begin{pmatrix} -5 \\ -9 \end{pmatrix}$ by their inverse to solve the equations. Very few attempted to post-multiply.

Answers: **(i)** $\frac{1}{8}\begin{pmatrix} 6 & -2 \\ -8 & 4 \end{pmatrix}$ **(ii)** $x = -1.5,\ y = 0.5$

Question 9

(i) This was answered well with the explicit use of the product rule evident. However, there was often additional work by many candidates cancelling $x \text{ and } \frac{1}{x}$ to give zero which affected their answer to part **(ii)**

(ii) Many did not see the link between this part and the previous part and even those that did were often let down by their poor manipulation of terms. Others attempted to integrate from scratch and obtained incorrect results such as $\frac{1}{2}(\ln x)^2$.

(iii) Many managed the first mark for using their answer to part **(ii)** correctly but further progress was often hampered by candidates not showing sufficient detail in their steps to obtain the given expression. There were a number who obtained a correct result from an incorrect start due to considerable 'fudging'. Some candidates did not attempt this part.

Answers: **(i)** $\ln x + 1$ **(ii)** $x\ln x - x + c$

Question 10

(i) Some candidates did not know where to start with solving this cubic equation. The fact that there were just three terms appeared to encourage some to use the quadratic formula whilst others subtracted 1 from each side of the equation and factorised what remained. Those that did know how to start usually correctly found that (c – 1) was a factor and then usually used long division to find the quadratic factor. Occasional errors were made in the long division, the most common being a missing constant term. However, there were a considerable number of completely correct solutions.

(ii) Most candidates differentiated at least one term correctly with many getting both right. There were some who thought that the product rule was needed.

(iii) Of those who had correctly differentiated $\tan x$ most were able to replace $\sec^2 x$ with $\frac{1}{\cos^2 x}$ and rearrange to show the given result. Those that began by replacing $\sec^2 x$ with $1+\tan^2 x$ more often made errors during their simplification. Many candidates thought they had to differentiate their expression again and equate it to 7.

(iv) Very few candidates spotted the link between part **(i)** and the rest of the question and as a result there were few correct attempts at this part. Some tried to factorise the expression not realising that they had already done this. Their second attempt was rarely as good as their first and was often abandoned half way through.

Answers: **(i)** $c = 1, \frac{1}{2}, -\frac{1}{3}$. **(ii)** $\sec^2 x + 6\cos x$ **(iv)** $x = 0,\ 1.05,\ 1.91$

Question 11

(i) This was generally very well done. A number of candidates found the y intercept and a few did not clearly identify that the positive solution to the quadratic equation was the one required.

(ii) The most straightforward method was to equate line and curve to eliminate y then use the discriminant to find m. There were a large number of candidates who carried out the first step but then seemed unable to make any further useful work, often substituting a value for x (often 2 as this possibly looked correct from the diagram). Those who did use the discriminant correctly did not always take notice of the diagram and did not realise that the gradient had to be negative. It was also possible to equate gradients and thus eliminate both m and y leading to a quadratic in x which gave the x coordinate of B and could then be used to find m. There were many solutions which either found the turning point or worked with the coordinates of one of the intercepts of the line. There were several incorrect methods used which actually arrived at the value of $m = -1$.

(iii) This required the value of m found in part **(ii)**. Many candidates realised this and were able to gain some credit as a result, although full credit relied on all earlier work being correct. Substitution of m back into the quadratic from part **(ii)** was most prevalent but equating m to the gradient function of the curve was also possible.

(iv) Most candidates knew that integrating the curve would be useful in some way and did so accurately. After that there were some very good and concise solutions using appropriate limits of integration from A and B and finding the area of the correct triangle. There were a number of solutions which completed one part correctly but not the other. As in the previous parts, candidates would do well to consider the diagram especially when deciding whether coordinates are appropriate or not. For example, the standard method of integrating line-curve did not fit here as the limits would be different. This could be overcome by the addition/subtraction of numerous other regions but this became highly complicated and rarely worked.

Answers: **(i)** $(4, 0)$ **(ii)** $m = -1$ **(iii)** $(2, 6)$ **(iv)** $10\frac{2}{3}$

ADDITIONAL MATHEMATICS

Paper 403723
Paper 23

Key messages

The number of marks available for a question is indicative of the amount of work needed to answer it. A question carrying only one or two marks should not require a long answer with a large amount of working.

Working should always be shown so that marks for method can be awarded, even when an answer is incorrect. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working.

For problems involving trigonometric expressions in calculus applications, any angle found, or any angle substituted, must be in radians, not degrees.

General comments

The overall quality of work seen varied greatly, with total marks covering the whole of the mark range. Some candidates showed an impressive amount of mathematical knowledge and skills. Others had insufficient knowledge for mathematics of this standard.

The marks allocated to each question or part question are shown throughout the question paper. This should be an indication to the candidate of the amount of work that is needed to provide an adequate answer. There were instances in this paper of far too much unnecessary work being done in relation to the marks available (see **Questions 5(iv)** and **5(v)**, and **Question 8(iv)** below).

In 'show that...' questions it is necessary to show every step in a proof, without combining steps and going directly to the given answer. Generally, for the three such questions on this paper, candidates performed well, with full step by step explanations being clearly provided.

It has been stated previously in these reports that in solving quadratic equations the method should always be shown, whether this is by factorisation or the use of the formula, but this message needs to be reiterated. If the equation is incorrect, a method mark can then be awarded. Method marks are not awarded for solutions to an incorrect equation taken directly from a calculator (see **Question 6(iii)** below).

Comments on specific questions

Question 1

In **part (a)** almost all candidates produced at least one correct diagram. Some seemed confused over what was required in **part (b)**, apparently not understanding the notation of the question.

Answer: **(b)** 18, 11, 29

Question 2

There were many full-mark answers to this question, but often more working than required was seen. Instead of solving just two equations, many candidates solved four, the second pair being exactly the same as the first pair, but in a different form. The most limited answers presented only the $x = 3$ solution. Candidates who squared both sides of the equation and solved the resulting quadratic almost always obtained both answers correctly.

Answer: $x = -1$, $x = 3$

Question 3

The process of rationalising the terms on the left-hand side of the equation was very well understood. Having done this, overall success was usually achieved by those candidates who knew the method of comparing surd and non-surd parts on the two sides of the equation. Those who did not, stopped at the point of having cleared the surds from the denominators, not knowing how to proceed.

Answer: $p = 5$, $q = 2$

Question 4

Good answers to this question converted the two logarithmic equations into simple linear equations, using the laws of logarithms correctly, and then solved them. Others used incorrect logarithmic relationships, or treated $\log_3$ as though it was in itself an algebraic entity, and produced longer attempts with scarcely any valid mathematics.

Answer: $x = 14$, $y = 5$

Question 5

The method of comparing the component parts of vectors on the two sides of an equation, in **part (iii)**, was the key to success here. Candidates who knew this method, and applied it correctly, frequently obtained full marks on the whole question. Many candidates answered **parts (i)** and **(ii)** correctly, but after setting up the equation in **part (iii)** stopped, clearly not knowing what to do with it.

In **parts (iv)** and **(v)** a lot of work was often seen, out of all proportion to the one mark available in each part. All candidates also need to be aware that expressions containing the division of one vector by another, frequently seen in these parts, are mathematically invalid.

Answers: **(i)** $\overrightarrow{OX} = \lambda(1.5\mathbf{b} + 3\mathbf{a})$ **(ii)** $\overrightarrow{OX} = \mathbf{a} + \mu(\mathbf{b} - \mathbf{a})$ **(iii)** $\lambda = \frac{2}{9}$, $\mu = \frac{1}{3}$ **(iv)** $\frac{1}{2}$ **(v)** $\frac{2}{7}$

Question 6

In **part (i)** many candidates demonstrated that they understood the meaning of $f^2(-3)$, but many others evaluated $[f(-3)]^2$ instead. The most efficient solutions used a simple two-stage numerical approach; the least efficient obtained a general algebraic form for $f^2(x)$ before finally substituting the specific numerical value. Correct answers were obtained more often from the first of these approaches than the second.

Very good general understanding was shown in **part (ii)** of how to find an inverse function. However, in a few cases no actual manipulation on $g(x)$ was carried out, the lines of algebra presented making no progress towards $g^{-1}(x)$, simply returning instead to their starting point.

In **part (iii)** the way in which the composite function was to be formed was almost always understood, although there were often errors of detail. After setting $gf(x)$ equal to $\frac{8}{19}$, good answers cross-multiplied, simplified to a quadratic equation, and then solved it. If an algebraic error was made, a method mark was still awarded for solving the quadratic equation, provided the method of solving was shown. In other answers, following the set-up, instead of cross-multiplying some candidates made the very serious error of equating the numerators and/or denominators of the two sides. This resulted in substantial mark loss, even when, fortuitously, the correct numerical answers were obtained.

Answers: **(i)** 17 **(iii)** $x = -5$, $x = 1$

Question 7

Candidates generally showed awareness of the fact that no calculus was involved in **part (i)**, integration was needed in **part (ii)**, and differentiation in **part (iii)**. As a result almost all obtained at least some marks on the question. A very common error in **part (i)** was to give the solution to the equation in degrees. In **parts (ii)** and **(iii)** marks were sometimes lost due to incorrect coefficients of $\sin 2t$ being used. There were two other common sources of error in **part (iii)**: sometimes zero was substituted into the expression for acceleration, and sometimes a rounded value of t from **part (i)** was substituted. Candidates need to be aware of the dangers of premature approximation, especially in longer structured questions, where the use of, say, a two significant figure answer from one part can lead to inaccuracy in a later part which uses the earlier answer.

Answers: **(i)** 0.615 s **(ii)** 0.715 m **(iii)** $-5.66\,\text{ms}^{-2}$

Question 8

Good understanding was shown by candidates who used only velocities in **parts (i)** and **(ii)**, before connecting velocity and distance in **part (iii)**. Many answers were seen which earned the first six marks. There were far fewer fully correct answers to **part (iv)**. Long, often unsuccessful, methods were offered which filled the whole of the answer space. The question was worth only two marks, and as such the candidate might have paused to consider that only a small amount of work should have been required. In the question, for the time which had to be found, the word ‘state’ was used, itself indicating that it was possible to write down the answer directly (exactly the same as that obtained in **part (iii)**) with no working whatsoever. Long methods were still rewarded when correct answers were obtained, but must have resulted in much wasted time for the candidate.

Answers: **(i)** 70.5° **(ii)** $2.83\,\text{ms}^{-1}$ **(iii)** 17.7 s **(iv)** 17.7 s, 14.1 m

Question 9

Many good proofs were seen in **part (i)**, with all the steps clearly shown. The strongest were those stating the values of $\frac{\mathrm{d}u}{\mathrm{d}x}$ and $\frac{\mathrm{d}v}{\mathrm{d}x}$ before they were substituted into the quotient rule or product rule. Many also obtained two of the three marks available in **part (ii)**, so that a good proportion of the candidates earned at least half marks on the question. It was very common to see a mark dropped in **part (ii)** as a result of the y-coordinate being presented only as a decimal. Possibly many candidates did not understand the significance of the word ‘exact’ in the question, if indeed they noticed it at all.

To be successful in **part (iii)** it was essential to understand how the result from **part (i)** could be rewritten in integral form. Candidates who started by doing this obtained at least some credit for their answers. Those who started with the required integral scarcely ever earned any marks.

Answers: **(ii)** $\left(\mathrm{e}^{\frac{1}{3}}, \frac{1}{3}\mathrm{e}\right)$ **(iii)** $-\frac{1}{9x^3} - \frac{\ln x}{3x^3}$

Question 10

Proofs in **part (a)** were generally very well presented, with all the steps shown clearly. Provided the candidate knew how to combine the two fractions, success followed almost always, although occasionally the final mark might be lost through omission of the factorisation step. Many fully correct answers were seen also to **part (b)(i)**. The most efficient used the identity relating $\cot^2 y$ and $\operatorname{cosec}^2 y$ (given on page 2 of the question paper) to form and solve a quadratic equation in $\operatorname{cosec} y$.

It was in **part (b)(ii)** where marks tended to be lost. The most creditworthy attempts were those which worked correctly throughout in radians, expressed as multiples of π. Work in radians expressed as decimals sometimes resulted in inaccurate answers. There were also errors as a consequence of incorrect quadrants being identified for solutions. Working and solutions presented in degrees only were awarded no marks.

Answers: **(b)(i)** 30°, 150°, 199.5°, 340.5° **(ii)** $\frac{7\pi}{24}$, $\frac{11\pi}{24}$

Question 11

In **part (i)** candidates who used the information that the root $x = 3$ was repeated, and so used the fact that $(x – 3)^2$ was a factor of the cubic, usually went on to obtain full marks. Almost all recognised that f(3) had to be zero (designating the cubic function as f(x)), but this alone was not enough. There sometimes followed incorrect attempts to obtain a second equation, perhaps by setting f(–3) or f(6) equal to zero. Such attempts earned no marks.

There were few fully correct answers to **part (ii)**. Much working was seen, often fruitless, focussed on finding a and b, which were not required, or using the same values for a and b as in **part (i)**. Roots were presented which were not repeated and not positive. Only the most perceptive candidates realised that the question simply required, in essence, three integers (other than 3, 3, 4) with a product of 36, one of the integers being repeated.

Answers: **(i)** $x = 4$, $a = –10$, $b = 33$ **(ii)** $x = 1$, $x = 1$, $x = 36$; $x = 2$, $x = 2$, $x = 9$; $x = 6$, $x = 6$, $x = 1$